Complexity and Distortion Analysis on Methods for Unrolling 3D to 2D Fingerprints

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***Abstract*— Fingerprint recognition systems have important applications for privacy and security. This has led to more studies and technologies that improve on the security and accuracy of fingerprint identification and verification systems. Such improvements and studies involve the application of three-dimensional (3D) fingerprint systems, where the details of the finger are captured using 3D technologies and the captured 3D fingerprints are converted into two-dimensional (2D) fingerprints. This paper presents a brief survey on different methods that are used to unwrap 3D fingerprint images into 2D fingerprint images, compare them on the effects of distortion and analyses two promising methods. The aim of this survey is to investigate complexity and properties of non-parametric methods so that they can be easily applied on different fingerprint identification and verification systems.**

Keywords*— *Fingerprints, Unwrapping, Unrolling, 3D fingerprints, 2D fingerprints.

I. Introduction

Fingerprint recognition systems are mostly used for identifying and verifying individuals. Most of these systems use touch based scanners that capture fingerprints. However, there are challenges encountered when matching fingerprints of the same finger that are captured in different instances or conditions. Some fingerprint images are affected:

- The deformation due to different pressure level, as shown in Figure 1,
- acquired at different times,
- small overlap,
- and the different dryness levels, as shown in Figure 2.

Fig. 1. Representation of different pressure applied during fingerprint acquisition

Fingerprint images in Figure 1 and Figure 2 were captured using a Futronic surface scanner. It can be seen that the fingerprint which was too dry (Figure 1a) missed some information due to incorrect ridge breaks; while the fingerprint which was too wet (Figure 1c) contains additional false information. Figure 1b) contains the ideal level of moisture.

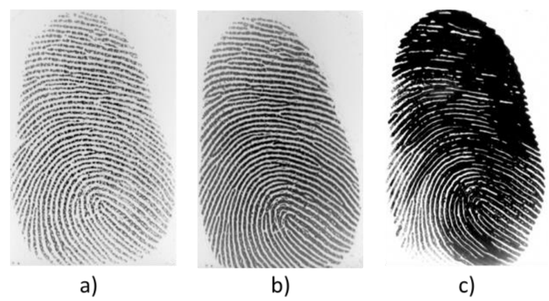

Fig. 2. Representation of fingerprint images captured at different instances with different levels of dryness and moisture, a) too dry, b) moisturised, c) wet.

Mostly used characteristics of fingerprints are minutiae point features. This is because; forensic examiners have relied on minutiae to match fingerprints for more than a century and expert testimony about suspect identity based on mated minutiae is admissible in a court of law [1]. Minutiae points are locations on the fingerprint where a ridge either terminates or divides to form two ridges; these are known as a ridge ending and a ridge bifurcation respectively.

To overcome the mentioned challenges, there is growing interest in the use of three-dimensional (3D) representation of fingerprints in fingerprint recognition systems, where 3D fingerprints are captured using scanners or technologies that capture a finger and represent it in a 3D model [1]. For legacy support, since most of existing fingerprint recognition databases store two-dimensional (2D) fingerprint images, it is important to have methods and technologies that unwrap 3D into 2D fingerprint images [2]. In addition, there is also the issue of matching two 3D fingerprints that may have been captured at different angles. Effective unwrapping is required to maintain the correct ridge distances so that two fingerprints captured in 3D may be accurately matched [3]. Unrolling or unwrapping is a method where 3D samples are mapped into the 2D space using algorithms that preserve the distances between discriminative points of the ridge pattern, as shown in Figure 3.

In [2] a survey of on 2D and 3D fingerprint technologies is presented, which is based on touchless technologies used to capture 3D fingerprint images. In addition, in 2011 [6] parametric models were compared with the non-parametric approach presented in [4] and [6]. In this report, the focus is on existing algorithms used to unwrap 3D fingerprint images, for the purpose of matching them with surface scanned 2D fingerprint images. The main aim of this work is to study how unwrapping algorithms deals with distortions both from unwrapped 2D and surface scanned 2D fingerprint images, and analyse the complexity of the non-parametric algorithms. In the following sections, a literature survey of various 3D unwrapping algorithms is presented.

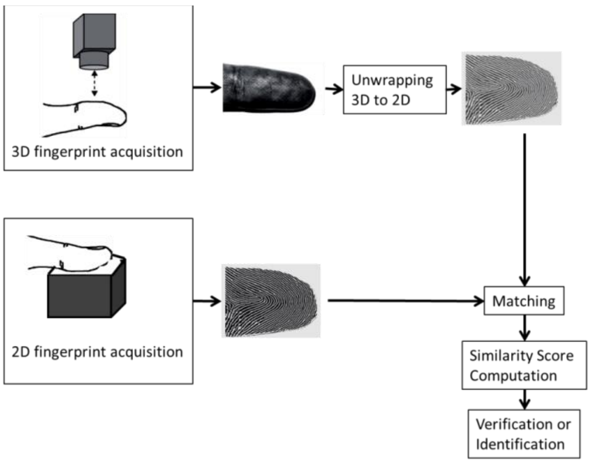

Fig. 3. Fingerprint matching system from the 3D fingerprint unwrapping to 2D [4].

II. UNWRAPPING ALGORITHMS

Unwrapping algorithms in the literature can be divided into parametric and non-parametric methods. Parametric methods are techniques that use defined geometrical models to approximate the finger shape. In these methods 3D fingerprints are projected to a parametric model (i.e., cylinder, tube, or sphere) [4]. On the other hand, non-parametric methods are techniques that do not perform assumptions on the finger shape, and to keep local distances and orientations of pixels, unwrapping is directly applied to the finger. Non-parametric methods are normally utilised for irregular-shaped objects, and the finger is assumed to have an irregular shape for the purpose of these methods [5].

A. Parametric unwrapping methods

A simple unwrapping algorithm based on a parametric strategy is presented in [4], [6] and [17], which it is based on the conversion of the 3D fingerprint model (x,y,z) into cylindrical coordinates (r,θ), as shown in Figure 4, where:

- r is the distance of the vector projected onto the (x,y) plane from A to B,
- θ is the angle between the projection of the vector onto the (x,y) plane and the positive X-axis, which is in the range of ($0 \leqslant \theta < 2\pi$),
- z is the Z-coordinate from the (x,y,z) model.

Figure 4 represents the conversion of point A in (x,y,z) coordinates into point B in (r,θ) coordinates. The similar method of converting each pixel in the (x,y,z) coordinate is applied when unwrapping a 3D fingerprint to equivalent 2D, as shown in Equation (1).

$$\begin{bmatrix} r \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} \sqrt{x^2 + y^2} \\ \tan^{-1}(y/x) \\ z \end{bmatrix} \quad (1)$$

The mathematical transformation is applied to fingerprint images by involving the projection of the 3D fingerprint onto the cylinder which takes the shape of the finger, and the 2D fingerprint is then obtained by flattening the cylinder. The cylinder model assumes that all slices are segments of circles that have the same radius. However, it is observed that the human fingers have different shapes when looking at each finger starting from a little finger to a thumb. The challenge on other fingers is that towards the fingertip they become narrow, which may cause distortion when the cylinder model is used. One of the shortcomings of this approach is that it does not preserve the relative distance between the points on the fingerprint surface, which introduces a horizontal distortion to the flattened fingerprint.

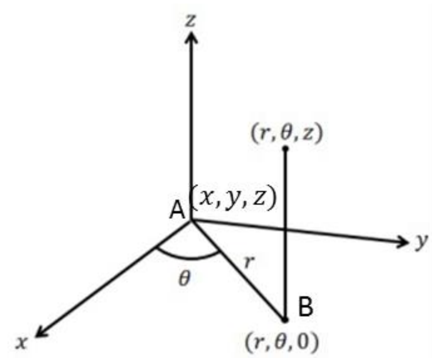

Fig. 4. Conversion from (x,y,z) coordinates to (r,θ,0) cylindrical coordinates

To reduce the computational cost and distortion caused by flattening the cylindrical model, the fit-sphere algorithm was presented [7],[16]. In this algorithm, the 3D fingerprint is projected to a best fitting spherical model by converting (x,y,z) coordinates to spherical coordinate (ρ,θ,ϕ), as shown in Figure 5, where:

- ρ is the length of the vector,
- θ is the angle between the positive Z-axis and the vector in question, with the range of ($0 \leqslant \theta \leqslant \pi$),
- ϕ is the angle between the projection of the vector onto the XY-plane and the positive Z-axis, with the range of ($0 \leqslant \phi < 2\pi$).

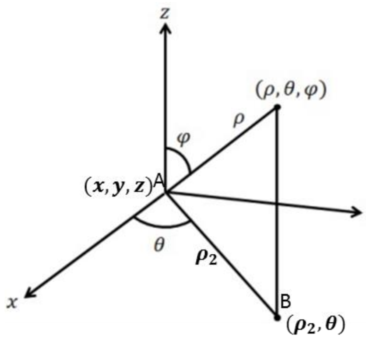

Fig. 5.Conversion from (x,y,z) coordinates to (ρ,θ,0) spherical coordinates

The similar method of converting a point in the (x,y,z) coordinate is applied, as illustrated in Equation (2).

$$\begin{bmatrix} \rho \\ \theta \\ \varphi \end{bmatrix} = \begin{bmatrix} \sqrt{x^2 + y^2 + z^2} \\ \cos^{-1}(z/\rho) \\ \tan^{-1}(y/x) \end{bmatrix} \quad (2)$$

A θ and ϕ projection was then created so that the 3-D fingerprint surface was mapped onto a 2D plane with minimal distortion. Finally after mapping to spherical coordinates, fingerprint ridges are extracted from depth by applying a band-pass filter to the Y-dimension where the low-frequency, smooth contours of the finger surface as well as the high-frequency, introduced noise is then removed. Since this algorithm is developed from parametric unwrapping methods, the computational cost is reduced compared to the cylindrical based method and, by taking advantage of detailed 3D information, the unwrapping based on the non-linear mesh achieves less deformation. However, the fit-sphere model introduces some distortions on the regions of the finger where the shape is not spherical, since the curvature changes throughout the finger. In [16] they attempted to solve this issue by applying a fitting low pass filter. The results from a low pass filter shows distinctive information from ridges and valleys of the fingerprint. That distinctive information is then removed and filtered out.

A flattening algorithm which unravels a deformable cylindrical tube has also been presented [8] and [10]. The aim of this algorithm is to overcome the challenge that the best-fit cylinder model has difficulty in capturing the fingerprint size variability in both horizontal and vertical directions, and that the finger is more tubular than spherical in shape. This method approximates the finger shape using a set of rings and then converts every ring in polar coordinates. The tube model assumes that all slices are segments of circles with different radii.

To form a tube fit to the 3-D surface, a spring based method is used to fit a single circle to a 2-D set of coordinates (x,y) for all points [11]. After defining how to fit a single circle to the fingerprint, the ring fitting process is then extended to form a tube by applying the single ring fitting to all positions along the surface in the Y-direction. It is assumed that the surface is cropped and holes filled such that there is only one contiguous surface. The last step in this method consists in the transformation of the fitted model into the 2D space. The coordinates of each column x of the slices in Y- and Z-coordinates are transformed into polar coordinates and the corresponding texture values related to the texture map are used to compute the unwrapped 2D fingerprint image by applying a linear interpolation.

An alternative approach which involves the analysis of the fingerprint curvature to determine the parametric shape was recently presented by Dighade [12]. In this method the convex hull is computed over the 3D fingerprint surface, and then the shape of the surface is used as a parametric model. The texture unwrapping is then applied to unwrap represented surface of the 3D fingerprint into 2D equivalent. As the parametric approaches it is important to preserve the relative distance between fingerprint features. In this method this is performed by using texture unwrapping through the multi-dimensional scaling technique [12]. In addition, parametric approaches intend to preserve angles to reduce the effect of deformation that occur when unwrapping to a 2D fingerprint, the use of multi-dimensional scaling leads to prevention of such distortion. As a result shown for this method, the unwrapped 2D fingerprint shows less stretch deformation and hence obtains higher accuracy in 2D equivalent fingerprints in matching surface scanned 2D fingerprint images over other unwrapping algorithms [12].

Compared to non-parametric methods, these methods are simple and straightforward transformation. However, these methods have shortcomings that the model must match the shape of a finger to avoid large distortions that can be introduced during unwrapping. In general, the shape of the finger is different from the tip to the first knuckle joint, as well as from the centre to nail-to-nail sides. Therefore, these methods allow some distortions to occur during the flattening of the model. These methods do not preserve the relative distances between points on the finger surface [12]. In addition, when the results of the unwrapped 2D fingerprint using these methods are matched with a corresponding 2D surface scanned fingerprint, it results in a high rejection error rate [8]-[12].

B. Non-parametric unwrapping methods

Lately in 2013 [1], a non-parametric approach on unrevealing equivalent 2D fingerprint from a 3D fingerprint was introduced. This method extracts the information of ridges and valleys to represent a 2D fingerprint. The first step in this method is to analyse captured 3D fingerprint information by determining the geometric shape on the fingerprint surface. Since some noise and distortion is introduced when capturing the 3D fingerprint, the next step is to detect the lines where ridges and valleys flow. Then detected ridges and valleys are highlighted so that the information that is not detected (which is deemed as noise) can be filtered. 2D equivalent is obtained by extracting the ridge and valley information from the 3D fingerprint and analysing the curvature of ridges.

A non-parametric method proposed by Chen et al.[4] locally unfolds the finger surface. Firstly a 3D finger is represented in the triangular mesh, where the vertices of triangles naturally form slices at different heights of the finger. Linear interpolation is applied in the vertical direction to obtain more slices between the given slices which results in a smoother unwrapped fingerprint. After a finer representation in the vertical direction is performed, a linear interpolation is then applied in the horizontal direction such that the neighbouring points of the same slice would correspond to neighbouring columns of the same row in the final unwrapped image [4]. The regeneration of the point for unwrapping starts from the centre and goes to the nail side. This non-parametric method generates better results than the parametric method since it preserves the relative distance between minutiae in the fingerprint [4]. Although the unwrapped fingerprint images using this method give better results than parametric methods, the rejection error rate is very high when the results are matched with corresponding surface scanned 2D fingerprints. This is because the unwrapped fingerprints are touchless, or deformation free, whereas the legacy rolled fingerprints involve noticeable skin deformation caused by the rolling.

Although presented algorithms perform unwrapping to some extent, 2011 Zhao et al. [6] improved on Chen et al.'s method [4] by introducing a new approach that specifically solves the problem of distortion on 2d equivalent images. This is performed by using direct sampling that change according to the curvature of the finger. The idea on this method is to compute equivalent non-uniform sampling rates on a 2D fingerprint to the non-uniform pressure applied on the scanned 2D fingerprint.

The method by Zhao et al. [6] attempt the general causes of distortion, which are due to the location, orientation and contact of the fingerprint to the surface scanner, during the acquisition process. The authors' assumptions are that during the acquisition, as the direction of the finger is perpendicular to the surface of the scanner. The centre of the captured finger is defined by determining the region on the finger that touches the surface of the scanner first. Another assumption is that once the finger touches the surface of the scanner, there is no twisting, rotation or relocation occurs.

These expectations lead to the conclusions that the pressure reaches the maximum at the centre and gradually decreases as the finger approach the edge of the surface scanner. Similarly, the sampling interval gradually increases from the centre to the edge. Although some distortions were resolved using this problem, this method introduced some distortions, because the distortion due to pressure is not linear throughout the finger because of the curvature of the finger. With this assumption, the size of ridges at the centre of the finger is not the same as the size of the ridges away from the centre. As shown in Figure 6, the ridges are wider at the centre compared to the tip of the finger, these images were captured using Futronic 2D surface scanner.

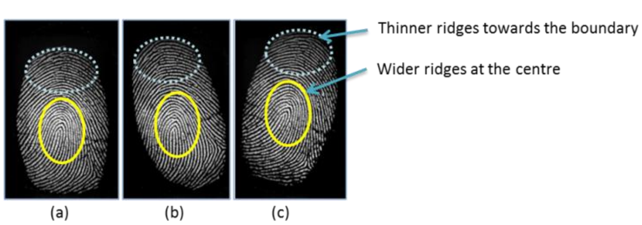

Fig. 6: Representation of the surface scanned finger with wider ridges at the centre.

In 2006 Fatehpuria et al [11] and Shafaei et al [12] introduced a new unwrapping method using the Springs Algorithm presented in [11]. In general image processing, the first step in springs algorithm is to identify regions where post-processing can be performed. The next step is to re arrange pixels per-sections of the identified regions to compute more similar distributions of pixels. Fatehpuria et al [11] proposed an algorithm that acquires a 2D rolled equivalent fingerprint image from a non-contact 3D finger scan.

The first step in this method is to extract the smooth surface of the 3D fingerprint by smoothing the ridge and valleys by a weighted, non-linear, least square algorithm [11]. Then, Gaussian function is used to compute a weight which smooths neighbours of the pixels that represent ridges and valleys. Fundamentally, this method treats each pixel point as if it is connected to its neighbouring points. This is performed by applying springs algorithm and to move it to a location where the energy in the springs is a minimum. The connected neighbouring points give smoother representation of the 3D fingerprint, which leads to computation of the equivalent 2D fingerprint. The rectangular mesh is then generated from connected nodal points. The rectangular has a relaxation distance generated to be equal to the Euclidean distance between two points in the 3D fingerprint [11].

The obtained smooth surface is then fitted into the rectangular mesh. The 2D surface is obtained by projecting the nodal points to a 2D plane and iteratively expanding them so that the total energy built can be reduced for each spring. The difference surface between the original 3-D fingerprint and the smoothened pixels, is then wrapped onto the resulting nodal points, which is then interpreted as the equivalent rolled 3-D fingerprint [11]. Although the results are better than the parametric methods, the distortion on the surface of the finger cause the representation of the unwrapped ridges to be unclear. This introduces false minutiae points and false information for fingerprint matching.

In 2009 Shafaei et al [14] further improved on the algorithm of Fatehpuria et al's [11] by involving the analysis of the curvature of 3D fingerprints. This was performed by using the curvature analysis from Gaussian and mean methods to determine the pixels that are on ridges and valleys from the unrolled 2D fingerprint surface. The Gaussian and mean method are used to determine the location and orientation of pixel flow in the ridges and valleys. After determining if the pixel is in the ridge or valley on the 3D fingerprint, then points that are on the ridge are replaced by a black colour on the unrolled 2D fingerprint.

The results of this approach showed better distortion throughout the finger, except on the regions where the finger surface had dip holes or cuts. When the results are matched with the corresponding 2D scanned fingerprints, the rejection rate is medium because the curvature of some regions is not analysed correctly due to pot-holes in a finger surface [12].

III. DISTORTION ANALYSIS

Here we discuss the findings and comparisons from the study according to different distortion conditions. The results of distortions were analysed from the presented algorithms and images. In Table I and Table II different types and causes of distortions are presented. The distortions that occur on the surface scanned 2D fingerprint are presented in Table I, and then the distortions on a 2D fingerprint caused by the process of unwrapping are presented in Table II.

TABLE I. DISTORTION CONDITIONS ON A SURFACE SCANNED 2D FINGERPRINT

Distortion	*Cause*
1. Unequal ridge size	Uneven Pressure on the Surface of the scanner
2. Connected ridges	Wet and too moisturized fingers
3. Displacement of minutiae	Pressure, different orientation and location of the finger
4. Unequal number of minutiae	Missing or additional minutiae from different regions of a finger

It can be seen in Figure 7 and Figure 8 that the distortion introduced during the process of unwrapping has effect on ridges of fingerprints. Methods in Figure 7 do not preserve the relative distances between points on the finger surface. As a result, the locations and orientations of minutiae points changes. This can lead into false matching when matching unwrapped image with the 2D surface scanned image.

TABLE II. DISTORTION CONDITIONS ON AN UNROLLED 3D FINGERPRINT

Distortion	*Cause*
1. Unequal ridge size	Due to barrel and pincushion distortions occur during unwrapping
2. Displacement of minutiae	Different orientation and location of the finger towards 3D fingerprint scanner
3. Unequal number of minutiae	Missing or additional minutiae from different regions of a finger
4. Connected Ridges	Finger moves while it is captured

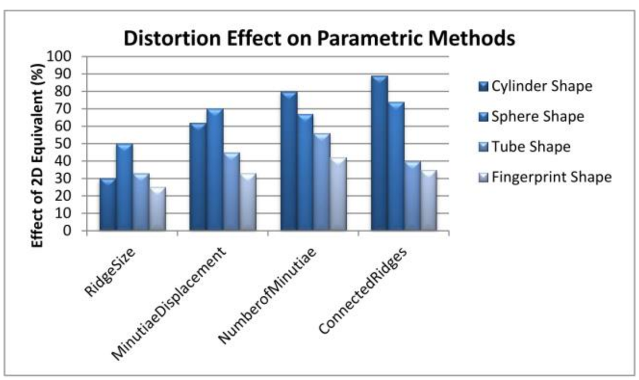

Fig. 7. Distortion effect on unwrapped 2D per parametric method

The 2D equivalent from the methods in Figure 8 gets affected when the size of ridges is not interpreted correctly. In addition, the results of these methods shows better distortion throughout the finger, except on the regions where the finger surface had dip holes or cuts. When the results are matched with the corresponding 2D scanned fingerprints, the rejection rate is medium because the curvature of some regions is not analysed correctly due to pot-holes in a finger surface.

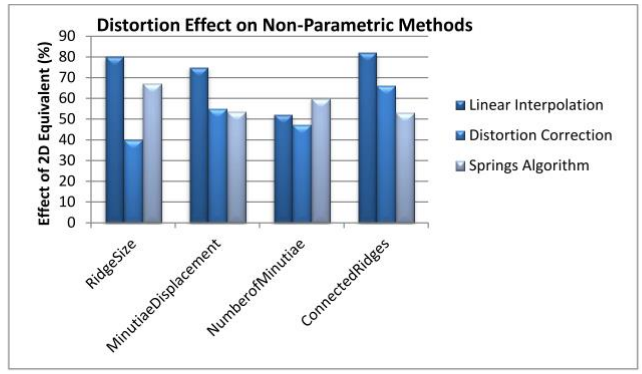

Fig. 8. Distortion effect on unwrapped 2D fingerprint as per non-parametric method

IV. COMPLEXITY ANALYSIS

Complexity analysis using big O notation involves the use of high-level description of the algorithm instead of an implementation of the pseudocode. The advantage of this type of analysis is that it takes into account all possible inputs of 3D fingerprints and their size, irrespective of the technology used to capture the image. In addition, it allows the evaluation of how fast is an algorithm independent of the hardware or software environment the algorithm can run on. During this research, pseudocodes of the algorithms in [1], [6] and [14] were inspected; this was performed to determine number of operations occurred in the algorithm as a function of the input size.

The promising methods are non-parametric method presented in [1], [6] and [14]. In this section presented is the complexity analysis of running time for these methods. The algorithms were constructed form the original articles. The analysis was performed using the big O notation. The results were then plotted using big O notations on Matlab.

In [1] 2D fingerprint is computed by extracting the information of valley-ridge lines. This process involves four steps, namely; determining fingerprint curvature, detecting ridges and valleys, filtering noise and outliers, and extracting ridges. The first step involves moving least square method which is used to estimate the surface shape of a fingerprint. The second step involves more computation because cross-correlation coefficient estimation is performed in order to filter noise and outliers. The time complexity of this method depends on its computational complexity which was calculated to be $O(n^3 \log n + n^2)$.

The method in [6] involves three steps, namely; vertical slice sampling, horizontal slices re-sampling, and unwrapping. The first step involves linear interpolation, which can be described using the following algorithm.

Initialize stepsize h, slicesize, verticalsize, Y_0,D_y
Initialize required variables
for *each slice* S_i *in* $3DFingerprint$ **do**
  for *each pixel* p *in slice* S_i **do**
    $$D_y = (p - Y_0) * h$$
    for *each stepsize* h_j *in* h *step though distance* **do**
      $$distance_pixel = Y_0 + D_y$$

The second step is presented on [4], with each slice being re-sampled horizontally. The last step involves the straight forward process of unwrapping from a resampled data. The main steps are the first two, which are analysed using the big O-notation. From the analysis, the complexity of the first and second step is O(n^2*(m+n)) + O(n*m), respectively. Where n is the number of slices and m is the size of each slice.

The third algorithm in [14] involves three steps, namely; linear regression springs algorithm, and unwrapping. The first step is presented in [15] which details the use of linear regression using Gaussian function. The second step is briefly detailed in the following algorithm. The complexity of this algorithm is O(n log n) + O(n^2).

Initialization $3D$ fingerprint image Max_x,Max_y, and Max_z
for *each pixel* (x_i, y_i, z_i) *in a 3D Fingerprint data* **do**
  If pixel is inside a matrix, toggle the value of (x_i, y_i, z_i) Store
  $best_{length} = current_{length}$
  for *each pixel* (o_j, p_j, q_j) *in surface* $S(x, y, z)$ **do**
    If pixel (o_j, p_j, q_j) is in the $3D$ data swap pixel (o_j, p_j, q_j) and $S(x_i, y_i, z_i)$ update
    $best_{length} = current_{length}$
  Update the magnitude of the springs between pixels in $S(x, y, z)$ and $Fingerprint(x, y, z)$ according to the best change

It can be observed from Figure 9 that the complexity of the algorithm presented in [1] uses more operations than other methods to produce accurate equivalent 2D fingerprint. This shows that the more complex the algorithm is results in better accuracy, however time complexity increases. An algorithm in [14] takes more operations than that presented in [6]. This is because of the iterations occurring on the linear regression method and springs algorithm. On the other hand, the algorithm in [6] shows less complexity.

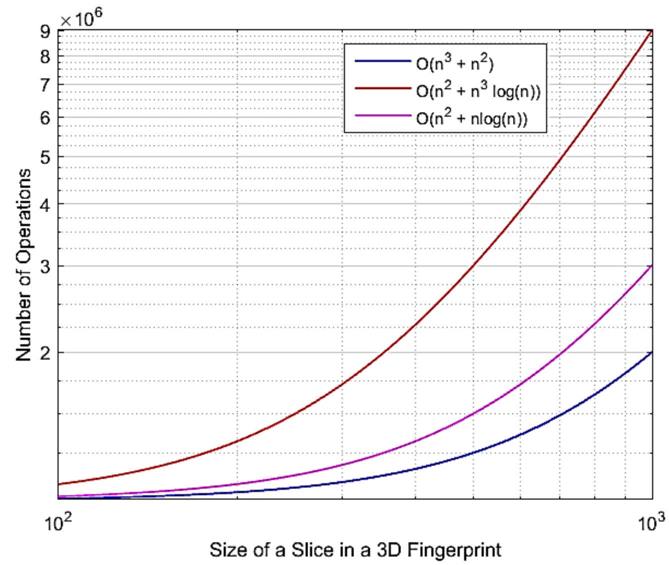

Fig. 9. Computational complexity of non-parametric algorithms

V. CONCLUSION

The important ideas which have been proposed for increasing the modelling representation of 2D fingerprint images under different conditions of distortions have been discussed in this paper. These include different methods of unwrapping fingerprint images that are captured using 3D scanners or technologies to present them as 2D fingerprint images. The consideration of 3D fingerprint unwrapping methods has led to the development of improved fingerprint identification and verification systems, where fingerprints from 2D surface scanners are identified or verified using 3D fingerprints. In addition, efficient implementations of unwrapping algorithms have been analysed and presented, by determining the fastest algorithm in unwrapping 3D to 2D fingerprint images. The comparison is performed only on two non-parametric algorithms because they are promising in terms of distortion correction.

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